Fahrenbacher's Theorem I

For every integer ***b*** greater than or equal to three, there exists another integer ***a*** in the range (0, ***b***) such that the summation of consecutive odd integers from the ***a***th to the ***b***th odd is the product of two primes, ***p*** and ***q***. In fact, the sum of these two primes is exactly the even integer 2***b***.

Fahrenbacher's Theorem is derived directly from Goldbach's conjecture: for every even integer ***x*** greater than two, there exists two primes, ***p*** and ***q***, such that there sum is equal to ***x***.

Proof I

Let ***x*** be an arbitrary even integer greater than four. Then Goldbach's conjecture is true if

$$x = p + q.$$

Similarly, we can express this equation this way

$$x=(x/2 + a) + (x/2 - a),$$

because we know p and q are equidistant from ***x/2***. Therefore, we can right the product of p and q as so:

$$pq=(x/2 + a)(x/2 - a).$$

This can obviously be simplified:

$$pq=(x^2)/4 -a^2.$$

We know that ***x^2/4*** is an integer because two divides ***x*** (because ***x*** is even), and so 2^2=4 divides ***x^2***. Also, this means that any prime factors that ***x^2*** has will have an even power (because if ***y*** is a prime factor of ***x***, then ***y^2*** is a prime factor of ***x^2***), so ***x^2/4*** is itself a perfect square, ***b^2***.

$$pq=b^2 -a^2.$$

Also, every perfect square z^2 is the sum of consecutive odds from one to z . Ex:[$z^2 = 1+3+...+z$]. Then, let's define O(i) be the ith odd number. Then,

$$pq=\sum O(i) \text{ from } i=a \text{ to } b.$$

And , the sum of ***p*** and ***q*** equals ***x*** (***2b***), so the proof is complete.